PSC 272

Spring 2003

Answers to Practice Set #1

Question 1.

(a)

Column

		C	D
	C	6 , 4	4 , 6
Row	D	2 , 0	0 , 2

Nash Equilibrium: (C, D)

Dominant Strategies: Row – C; Column - D

(b)

Column

		C	D
	C	4 , 4	0 , 6
Row	D	6 , 0	2 , 2

Nash Equilibrium: (D, D)

Dominant Strategies: Row – D; Column - D

(c)

Column

		C	D
	C	6 , 6	0 , 4
Row	D	4 , 0	2 , 2

Nash Equilibria: (C, C) ; (D, D)

Dominant Strategies: none

(d) Column

		C	D
	C	4 , 4	2 , 6
Row	D	6 , 2	0 , 0

Nash Equilibria: (C, D) ; (D,C)

Dominant Strategies: none

(e) Column

		C	D
	C	6 , 4	2 , 2
Row	D	0 , 0	4 , 6

Nash Equilibria: (C, C) ; (D, D)

Dominant Strategies: none

Question 2.

Austria has three options:

- If withdraws➔ status quo
- If attacks➔Austria wins with p, loses with 1-p.
- If issues ultimatum➔Prussia resists with q, which means a war that Austria wins with p, loses with 1-p. Prussia acquiesces with 1-q.

(a) *EU(Attack) =Prob(win)*U(win)+Prob(lose)*U(lose)*

$=p*6+(1-p)*0$

$=6p$

Note that this is also Austria's expected utility of war, which occurs if Prussia resists. That is, *EU(War)=6p* for Austria.

(b) $EU(Ultimatum) = Prob(Prussia\ Resists)*EU(war)+Prob(Prussia\ Acq)*U(Acq)$

$=q*6p+(1-q)*10$

$=6pq+10(1-q)$

(c) Need to compare the expected utility from attacking with both the expected utility from ultimatum and withdrawal.

$EU(Attack)>EU(withdrawal)$

$6p>5$

$p>5/6$

So, if the probability of winning is greater than 5/6, Austria will attack rather than withdraw.

$EU(Attack)>EU(Ultimatum)$

$6p>0.5*6*p+10*(1-0.5)$

$6\ p>3p+5$

$p>5/3$

This means Austria would *never* choose to attack rather than issue an ultimatum. (It would only do so if *p>5/3, which is impossible* because, by the definition of a proper probability, 0< p<1.) Intuitively, if it issues an ultimatum its worst possible outcome is a war, and there is some chance that it gets what it wants without having to fight one. This completely answers the question. We might also want to know when Austria will feel confident enough of winning to issue an ultimatum, and we can find that the same way:

$EU(Ultimatum)>EU(withdrawal)$

$6pq+10(1-q) > 5$ note: q=.5

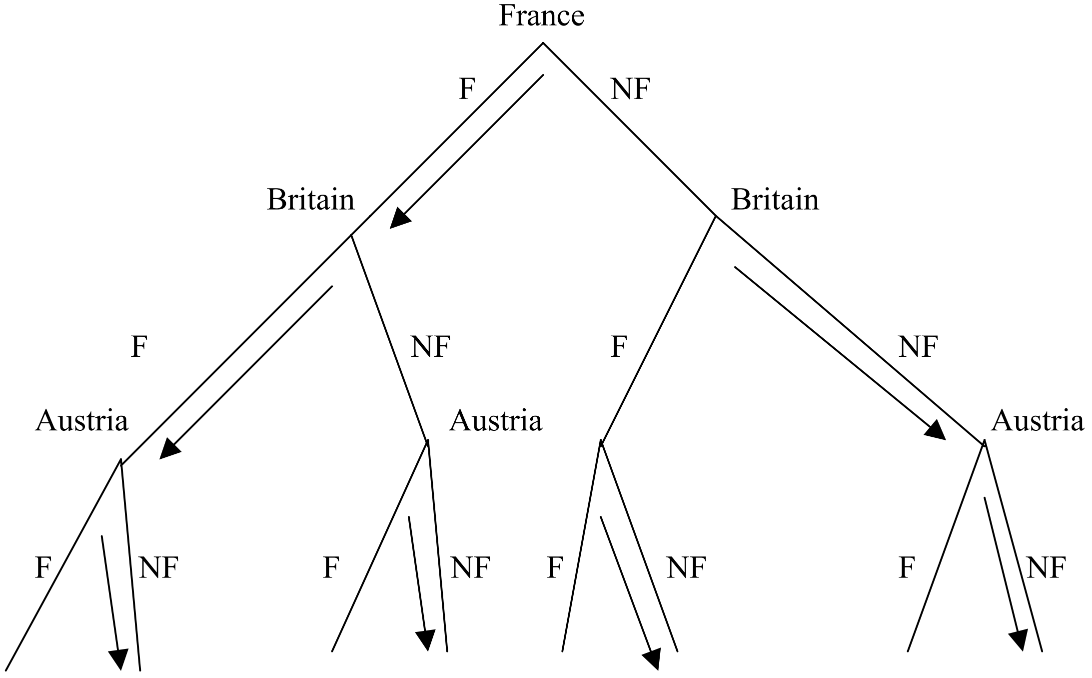
$3p > 5 - 10(.5)$

$p > 0$

So, if q is as low as .5, Austria is willing to issue an ultimatum no matter how likely it is to win. This is what happened at Olmuetz, and Prussia backed down (largely because Austria had support from Russia). This became known as the "humiliation of Olmuetz," which made Bismarck determined to defeat Austria and ultimately led to the Austro-Prussian War of 1866.

Question 3.

(a)



F	5	4	3	1	2	2	2	2
B	1	4	3	3	1	1	3	2
A	1	2	1	2	1	2	1	2

Subgame Perfect Equilibrium: (F, F, NF). That's essentially what happened in the Crimean War, although Austria did threaten to intervene to get Russia to withdraw from Rumania.

(b)

Austria

F

		Britain	
		F	NF
France	F	5, 1, 1	3, 3, 1
	NF	2, 1, 1	2, 3, 1

NF

		Britain	
		F	NF
France	F	4, 4, 2	1, 3, 2
	NF	2, 1, 2	2, 2, 2

Nash Equilibria: (F, F, NF); (NF, NF, NF)

(c) Extra credit: Sequential choices

Countries's strategies – specify a course of action for each information set.

France: {F, NF} (One information set x two actions= $2^1 = 2$ strategies.)

Britain: {F F (always fight), F NF (fight only if France does), NF F (fight only if France doesn't), NF NF (never fight)}. (Two information sets x two actions= $2^2 = 4$ strategies.)

Austria: Four information sets: sixteen strategies ($2^4 = 16$). Strategy examples: {F F F F, F F F NF, F NF F F, F F NF F, NF NF F F, ….}

The strategic form game can be drawn as follows: two big cells for France's two strategies, fight (F) not fight (NF); within each of these cells one would have a 16 * 4 table with the strategies of Austria and England, respectively. Find the payoffs to put in each cell by applying each strategy triplet to the extensive form of the sequential game, and finding which outcome it leads to.

This is pretty cumbersome, but remember that we can simplify a strategic form game by eliminating dominated strategies. Since Austria always prefers to remain aloof from the fighting, we can eliminate F F F F, which is dominated by NF NF NF NF. We can eliminate England's NF F strategy, because F NF dominates it. This gets the problem down to more manageable dimensions (2 x 3 x 15), but it's still a mess, because it is in equilibrium for Austria to fight under circumstances that it can safely assume won't arise: for example, England fights, France doesn't. There are eight Nash equilibria in which England and France fight and Austria abstains, and sixteen in which none of the powers intervenes.

Question 4.

(a)

Player 1	C	C	C	…
Player 2	C	C	C	…

If both players play Grim Trigger we get cooperation indefinitely. The utility for each player in this case is:

$$U(GRIM|GRIM) = \frac{R}{1-\delta}$$

Now consider a deviation to Always Defect (ALLD) by player 2, holding player 1's strategy fixed, i.e., player 1 still plays Grim Trigger:

Player 1	C	D	D	D…
Player 2	D	D	D	D …

Player 2's utility from such a deviation is:

$$U_2(ALLD|GRIM) = T + 0 \cdot \delta + 0 \cdot \delta^2 + \cdots = T$$

To find δ that induces deviation to ALLD:

$$U_2(ALLD|GRIM) > U_2(GRIM|GRIM)$$

$$T > \frac{R}{1-\delta}$$

$$\delta < \frac{T-R}{T}$$

Now consider a deviation to Grim DEV1L, which means deviating to ALLD at some period L:

Player 1	C	C	C	D	D	D…
Player 2	C	C	D	D	D	D …

↑

Period L

To calculate δ for this case, we only need to compare player 2's payoffs from period L on.

Player 2's utility of playing GRIM from period L on is:

$$U_2(GRIM|GRIM)\ \text{from period } L \text{ on} = \frac{R\delta^L}{1-\delta}$$

Player 2's utility of playing GRIM DEV1L from period L on is:

$$U_2(GRIM\ DEV1L|GRIM)\ \text{from period } L \text{ on} = T\delta^L + \sum_{t=L+1}^{\infty} \delta^t (0) = T\delta^L$$

To find δ that induces deviation to GRIM DEV1L:

$$U_2(GRIM\ DEV1L|GRIM) > U_2(GRIM|GRIM)$$

$$T\delta^L > \frac{R\delta^L}{1-\delta}$$

$$\delta < \frac{T-R}{T}$$

The required discount rate for GRIM DEV1L is the same as for ALLD.

(b)

Player 1	C	C	C	…
Player 2	C	C	C	…

If both players play TFT, then we get cooperation indefinitely. And,

$$U(TFT|TFT) = \frac{R}{1-\delta}$$

Now consider a deviation to Always Defect (ALLD) by player 2, holding player 1's strategy fixed, i.e., player 1 still plays TFT:

Player 1	C	D	D	D…
Player 2	D	D	D	D …

Player 2's utility from such a deviation is:

$$U_2(ALLD|TFT) = T + 0 \cdot \delta + 0 \cdot \delta^2 + \cdots = T$$

To find δ that induces deviation to ALLD:

$$U_2(ALLD|TFT) > U_2(TFT|TFT)$$

$$T > \frac{R}{1-\delta}$$

$$\delta < \frac{T-R}{T}$$

Now consider a deviation to DEV1 L, which means TFT until period L; then play D once, play C once and TFT thereafter.

Player 1	C	C	C	D	C	C…
Player 2	C	C	D	C	C	C …
			↑ Period L			

To calculate δ for this case, we only need to compare player 2's payoffs from round L and L+1 for TFT and DEV1L strategies, since for other periods the two strategies give exactly the same payoffs. Player 2's utility from period L and L+1 for playing TFT is:

$$U_2(TFT|TFT)\ \text{for}\ L\ \text{and}\ L+1 = R\delta^{L} + R\delta^{L+1}$$

$$U_2(DEV1L|TFT)\ L\ \text{and}\ L+1 = T\delta^{L} + S\delta^{L+1}$$

To find δ that induces deviation to DEV1L:

$$U_2(DEV1L|TFT) > U_2(TFT|TFT)$$

$$T\delta^L + S\delta^{L+1} > R\delta^L + R\delta^{L+1}$$

$$\delta < \frac{T-R}{R-S}$$

If R-$S < T$, then it is harder to prevent deviation to DEV1L. On the other hand, if the sucker's payoff is very bad so that $R - S > T$ (remember that S<0) , then it is easier to prevent deviation to DEV1L.

(c) Nash equilibrium and subgame perfect equilibrium

As we see in part (a) and (b), both GRIM and TFT are Nash equilibria for certain range of δ, in the sense that if one of the players plays GRIM (TFT), then the other player cannot do better by deviating to other strategies; her best response is GRIM (TFT) as well.

Subgame perfect equilibrium (SPE) is more restrictive than Nash equilibrium (NE). (All SPE are NE, but not all NE are SPE.) While NE only requires that a strategy is a best response in the whole game, SPE requires a strategy to be a best response in every subgame as well as in the whole game (actually, the whole game can be thought of as one subgame). GRIM is a SPE in the sense that it is a best response for a player in every subgame. To check this, consider what happens if my partner doesn't deviate. Do I want to? Not so long as my discount factor is high enough. What if my partner does deviate? Then my strategy calls on me to defect forever. Given that my partner is playing a GRIM strategy, my partner will also defect forever, regardless of what I do. Consequently, I have no incentive to deviate from my strategy.

TFT is not a SPE. In some subgames, a player is better off playing some other strategy than playing TFT. Think of an infinitely repeated game as an infinitely branching tree, where each branch represents a possible action and the infinite sequence of events that follows from it. If we can find one branch in which there is something better to play than TFT, given that my partner is playing TFT, then TFT is not subgame perfect. Remember that this branch may even be one that will never be reached as long as we keep playing our equilibrium strategies. Consider what happens on the equilibrium path: we both cooperate forever. Do I have an incentive to deviate? Not as long as my discount factor is high enough. Consider what happens if one of us deviates once. Since I'm playing TFT, if you deviate, I defect next period. Since you're playing TFT, you'll be cooperating while I'm defecting; but in the following period, you'll defect. That will provoke me to defect, etc. This is essentially what's happening in the Israel/Palestine dispute:

Player 1	C	D	C	D	C …
Player 2	C	C	D	C	D…

This series of events is worse than other strategies I could choose. For example, I could play "forgiving" TFT, and only punish if you deviate twice in succession; then your one-period deviation would only cost me one sucker payoff instead of an infinite alternation between T and S, and then we could go back to profitable cooperation. Perhaps someone should suggest this to Ariel Sharon?