

Political Science 272: Problem Set 1

Fall 2005. Due at 5:00, Friday, October 14. Please, not in my mailbox; slip it under my door.

Write your answers neatly on a separate sheet of paper (attach as many sheets as necessary). Show all calculations, game diagrams, etc. Partial credit will be given for getting part way to an answer. Please write your name on your work and staple the pages.

1. Strategic-form games.

Find and circle all Nash equilibria in pure strategies and cross out all dominated strategies.

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2. Dominated strategies

Solve the following game. The utilities in the cells are for (Row, Column). Circle all Nash equilibria. Cross out all strictly dominated strategies. **Hint:** Eliminate as many strategies as possible first and then iterate the elimination of dominated strategies.

	1	2	3	4	5	6
A	4, 3	4, 3	1, 1	-4, 4	3, 3	0, 2
B	-4, -6	1, 1	-4, 6	-4, 6	1, 1	1, 4
C	0, 3	3, 0	0, 0	2, 2	3, 1	0, 0
D	2, 2	2, 2	-6, 3	2, 4	0, 3	2, 3
E	1, 2	0, 0	3, 2	1, 5	0, 0	0, 3
F	2, 2	1, 2	2, 3	1, 1	4, 3	6, 0
G	3, 3	2, 2	6, 3	6, 3	6, 2	-2, 2
H	2, 2	4, 3	4, 4	2, 4	2, 2	2, 3
I	2, 3	-2, 3	1, 2	-4, 2	-4, 0	-4, 6
J	1, -6	1, 1	1, 1	0, 3	0, 0	-6, 6

First, compare several Row strategies to H; this allows us to eliminate B, E, and J. Next, compare I to G, and eliminate I. Now, compare 6 to 3, and eliminate 6. Now, compare F to G, eliminate F. (Notice, we couldn't do that until we had eliminated 6.) Next, compare 5 to 4, and eliminate 5. (Again, this only works after eliminating F.) Next, compare 2 to 4, and eliminate 2. Next, compare C, D and H to G, and eliminate C, D and H. This procedure leaves us with the following reduced game, which no longer contains any dominated strategies:

	1	3	4
A	4, 3	1, 1	-4, 4
G	3, 3	6, 3	6, 3

There are two Nash equilibria (circled above).

3. Lotteries.

George W. Bush wants to fight a war with Iraq, and prefers to do so with multilateral support. Should he ask the UN Security Council for a resolution authorizing the use of force? The political fallout from starting the war will be greatly reduced, particularly for Bush's ally Tony Blair, if there is such a resolution. However, asking for such a resolution and facing a rejection by the Security Council will increase the political cost of war. Bush estimates that the Security Council will approve a resolution with probability p , and that the war will have a satisfactory outcome (victory) with probability q . Bush's cardinal utilities for each outcome are as follows: $U(\text{Victory}|\text{Resolution})=4$; $U(\text{Victory}|\text{No request})=3$; $U(\text{Victory}|\text{Rejection})=2$; $U(\text{Defeat}|\text{Resolution})=-4$; $U(\text{Defeat}|\text{No request})=-5$; $U(\text{Defeat}|\text{Rejection})=-6$; $U(\text{Peace})=0$

- (a) Express Bush's expected utility of going to war if the UN supports him with a resolution

$$U(\text{WAR}|\text{Resolution})=q(U(\text{Victory}|\text{Resolution})+(1-q)U(\text{Defeat}|\text{Resolution}))$$

$$=4q-4(1-q)=8q-4$$

- (b) Express his expected utility of going to war without UN support

Sorry, this was ambiguous. I meant, the utility of war after a rejection, but I'll accept the utility after a rejection or the utility after no request.

$$U(\text{WAR}|\text{Rejection})=q(U(\text{Victory}|\text{Rejection})+(1-q)U(\text{Defeat}|\text{Rejection}))$$

$$=2q-6(1-q)=8q-6$$

$$U(\text{WAR}|\text{No request})=q(U(\text{Victory}|\text{No request})+(1-q)U(\text{Defeat}|\text{No request}))$$

$$=3q-5(1-q)=8q-5$$

- (c) Suppose Bush has requested a resolution and the Security Council has rejected his request. How high does the probability of victory, q , have to be for Bush to initiate a war?

$$U(\text{WAR}|\text{Rejection}) \geq U(\text{Peace})$$

$$q(U(\text{Victory}|\text{Rejection})+(1-q)U(\text{Defeat}|\text{Rejection})) \geq 0$$

$$8q-6 \geq 0$$

$$q \geq .75$$

(d) Suppose Bush requests a resolution and the UN grants one. How high does the probability of victory, q , have to be for Bush to initiate a war in this case?

$$U(\text{WAR}|\text{Resolution}) \geq U(\text{Peace})$$

$$q(U(\text{Victory}|\text{Resolution}) + (1-q)U(\text{Defeat}|\text{Resolution})) \geq 0$$

$$8q-4 \geq 0$$

$$q \geq .5$$

(e) Suppose the probability of victory is sufficiently high that Bush will go to war under any circumstances. How high does the probability of passing a resolution, p , have to be for Bush to take the issue to the Security Council?

$$pU(\text{WAR}|\text{Resolution}) + (1-p)U(\text{War}|\text{Rejection}) \geq U(\text{WAR}|\text{No request})$$

$$p(8q-4) + (1-p)(8q-6) \geq 3q-5(1-q)$$

$$p \geq .5$$

4. Mixed Strategy Nash Equilibrium

The numbers in the cells are cardinal utilities for (Row, Column). Solve for the mixed strategy Nash equilibrium.

		qC	$(1-q)D$
pC		4, -2	-2, 1
$(1-p)D$		-2, 2	3, -4

$$p=2/3; q=5/11$$

5. Repeated games.

Assume that two players find themselves in a repeated game of Prisoner's Dilemma (i.e., $DC > CC > DD > CD$ for each player, where the first letter indicates "my" strategy and the second "your" strategy), and each attaches the following cardinal utilities to the respective outcomes: $U(CC)=3$, $U(DC)=6$, $U(CD)=1$, $U(DD)=2$. The game is infinitely repeated, and each actor discounts future payoffs, using a common discount factor, δ .

(a) Find the Nash equilibrium of the stage game (a single-play of PD).

DD

- (b) Find the discount factor necessary to prevent a player from defecting if both players play *grim trigger* strategies: play C in the first period; always continue to play C if the other player does likewise, and always defect if the other player has ever defected.

$\delta \geq 3/4$

- (c) Does this pair of strategies form a Nash equilibrium? Does it form a subgame perfect equilibrium (SPE)? Why (or why not)?

Yes, it's Nash. On the equilibrium path, if $\delta \geq 3/4$, neither player has an incentive to defect. Yes, it's SPE. Again, on the equilibrium path, if $\delta \geq 3/4$, neither player has an incentive to defect. Off the equilibrium path, neither player ever has an incentive to play anything other than defect. GRIM calls for eternal defection off the equilibrium path, so the strategies (GRIM, GRIM) form a Nash equilibrium in every subgame off the equilibrium path as well.

- (d) Now find the discount factor necessary to prevent a player from defecting if both players play *tit-for-tat* strategies: play C in the first period; thereafter always play whatever the other player played in the previous period.

It does not exist (i.e., $\delta > 1$, which is impossible, so (TFT, TFT) is not a Nash equilibrium for any discount factor). The reason is that when one of us defects, we subsequently alternate between cooperation and defection, which is better than cooperation (this is true for all discount factors, because we always get to enjoy the unilateral defection before we have to suffer the consequences). This occurs in this example because $CC < CD + DC$.

- (e) Does this pair of strategies form a Nash equilibrium? Does it form a subgame perfect equilibrium? Why (or why not)?

It is not Nash, and therefore not SPE.